

AD-A065 862

FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO
ANALYTICAL PERTURBANCES IN AN UNSTEADY TRANSONIC FLOW, (U)
SEP 77 Y V MAMONTOV

F/6 20/4

UNCLASSIFIED

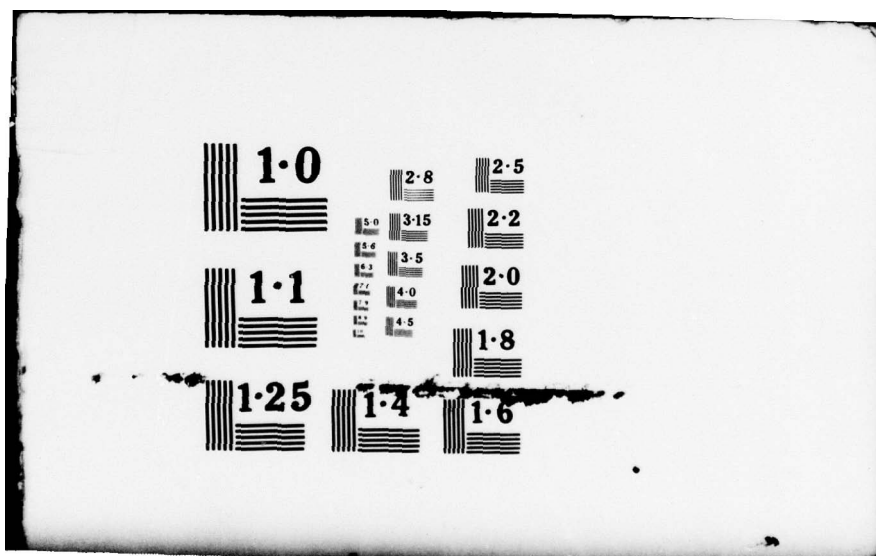
FTD-ID(RS)T-1579-77

NL

OF
ADA
065862



END
DATE
FILMED
4-79
DDC



AD-A065862

1

FTD-ID(RS)T-1579-77

FOREIGN TECHNOLOGY DIVISION



ANALYTICAL PERTURBANCES IN AN UNSTEADY TRANSONIC
FLOW

by

Ye. V. Mamontov



DDC
RECEIVED
MAR 19 1979
D

Approved for public release;
distribution unlimited.

78 11 09 114

EDITED TRANSLATION

FTD-ID(RS)T-1579-77 12 September 1977

MICROFICHE NR: *FTD-77-C-001182*

ANALYTICAL PERTURBANCES IN AN UNSTEADY TRANSONIC FLOW

By: Ye. V. Mamontov

English pages: 9

Source: Dinamika Sploshnoy Sredy, Novosibirsk,
No. 10, 1972, PP. 217-222

Country of origin: USSR

Translated by: Carol S. Nack

Requester: FTD/PDRS

Approved for public release; distribution unlimited

ADDITIONAL INFO	
DTIC	White Section <input checked="" type="checkbox"/>
DDC	Staff Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY STATEMENT	
Dist. STATE. UNCLASS. SPECIAL	
A	

<p>THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.</p>	<p>PREPARED BY:</p> <p>TRANSLATION DIVISION FOREIGN TECHNOLOGY DIVISION WP-AFB, OHIO.</p>
---	---

U. S. BOARD ON GEOGRAPHIC NAMES transliteration SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З э	<i>З э</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
 When written as ё in Russian, transliterate as yë or ë.
 The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	A α	Nu	N ν
Beta	B β	Xi	Ξ ξ
Gamma	Γ γ	Omicron	Ο ο
Delta	Δ δ	Pi	Π π
Epsilon	Ε ε	Rho	Ρ ρ
Zeta	Ζ ζ	Sigma	Σ σ
Eta	Η η	Tau	Τ τ
Theta	Θ θ	Upsilon	Υ υ
Iota	Ι ι	Phi	Φ φ
Kappa	Κ κ	Chi	Χ χ
Lambda	Λ λ	Psi	Ψ ψ
Mu	Μ μ	Omega	Ω ω

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
---------	---------

sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	\sin^{-1}
arc cos	\cos^{-1}
arc tg	\tan^{-1}
arc ctg	\cot^{-1}
arc sec	\sec^{-1}
arc cosec	\csc^{-1}
arc sh	\sinh^{-1}
arc ch	\cosh^{-1}
arc th	\tanh^{-1}
arc cth	\coth^{-1}
arc sch	sech^{-1}
arc csch	csch^{-1}

rot	curl
lg	log

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

ANALYTICAL PERTURBANCES IN AN UNSTEADY TRANSONIC FLOW

Ye. V. Mamontov

Small perturbances in an unsteady transonic gas flow are described by the equation [1], [2]

$$-\varphi_{xx} + \varphi_{yy} - 2\varphi_{xt} = 0 \quad (1)$$

Here $\varphi(x, y, t)$ (dimensionless) is the perturbation velocity potential. The variables x, y, t are also dimensionless.

The following boundary problem was formulated in [3], [4]. In the region $D = \{(x, y, t); t > 0, x > 0, -\infty < y < \infty\}$ it is necessary to find the solution to $\varphi(x, y, t)$ of equation (1), if we know

78 11 09 114

$$\phi(x, y, 0) = \phi_0(x, y), \quad \phi(0, y, t) = \phi_0(y, t) \quad \text{and} \quad \phi_x(0, y, t) = \phi_1(y, t) > 0$$

In other words, potential ϕ is known at the initial point in time throughout the flow, while the values of both components of the velocity vector are known on straight line $x = 0$ at $t > 0$. Inequality $\phi_1 > 0$ means that this straight line is in the supersonic part of the flow. The uniqueness of the solution to this problem and its validity for the linear model of equation (1) were shown in these reports.

From the physical standpoint, it is logical to anticipate that a solution will exist at small values of time t . This report studies the problem of the existence of analytical solutions.

This is based on the theorem proven by L. V. Ovsiyannikov [5] about the solvability of differential equations in the scale of Banach spaces. We will write the necessary equation, we will select the scale of Banach spaces, and we will check the solvability of the conditions of L. V. Ovsiyannikov's theorem.

1. We will consider equation (1) in region $D = \{(x, y, t) \mid T > 0, x > 0, -\infty < y < \infty\}$ and we will assume that the values of solution $\phi(x, y, t)$ are assigned in planes $t = 0$ and $x = 0$. Equation (1) can be replaced by the system

$$\begin{aligned} u_t &= -\frac{1}{2} uu_x + \frac{1}{2} v_y \\ u_y - v_x &= 0 \end{aligned} \quad (2)$$

where $u = \varphi_x$, $v = \varphi_y$ are the perturbation velocities on axes x and y , respectively. The values $u(x, y, 0) = u_0(x, y)$ and $v(0, y, t) = h(y, t)$ are found from the assigned values of the potential. Then

$$v(x, y, t) = h(y, t) + \frac{\partial}{\partial y} \int_0^x u(\xi, y, t) d\xi.$$

Substituting this expression in the first equation of the system, we will obtain the equation for function u

$$u_t = -\frac{1}{2} uu_x + \frac{1}{2} \frac{\partial^2}{\partial y^2} \int_0^x u(\xi, y, t) d\xi + \frac{1}{2} \frac{\partial h}{\partial y} \quad (3)$$

with the initial condition

$$u(x, y, 0) = u_0(x, y). \quad (4)$$

If function $u(x, y, t)$ is found, the potential can obviously be reconstructed. Using the substitution

$$w = u - u_0$$

we will give the initial condition for zero and we will consider the

problem

$$w_t = -\frac{1}{2} w w_x - \frac{1}{2} u_0 w_x + lw - \frac{1}{2} u_{0x} w + lu_0 - \frac{1}{2} u_0 u_{0x} + \frac{1}{2} \frac{\partial h}{\partial y}, \quad (3')$$

$$(x > 0, t > 0, -\infty < y < \infty),$$

$$w(x, y, 0) = 0, \quad (4')$$

where operator l acts as follows

$$lw = \frac{1}{2} \frac{\partial^2}{\partial y^2} \int_0^x w(\xi, y, t) d\xi.$$

2. The solvability of this problem will be proven in the analytical scale of the Banach spaces $S = U B_p$. Space B_p consists of

$$0 < p \leq p_0$$

of function $w(x, y, t)$, for which the norm

$$\|w\|_p = \sum_{n=0}^{\infty} \frac{1}{n!} |^nw|_p^n \quad (5)$$

is finite at

$$|^nw| = \max_{0 \leq k \leq n} |w_{k, n-k}| = \max_{0 \leq k \leq n} \left| \frac{\partial^n w}{\partial x^k \partial y^{n-k}}(0, 0, t) \right|.$$

These norms have the necessary properties [5]. We will also point out

that the following inequalities are valid:

$$\|w_1, w_2\|_p \leq \|w_1\|_p \|w_2\|_p, \left\| \frac{\partial}{\partial x} w \right\|_p \leq \frac{\partial}{\partial p} \|w\|_p \quad (6)$$

3. Problem (3'), (4') can be considered to be the problem of searching for the solution to equation

$$\frac{dw}{dt} = f(w, t) \quad (7)$$

with the initial condition

$$w(0) = 0 \quad (8)$$

The nonlinear operator acts in the scale of Banach spaces if we require that functions u_0 , u_{0x} , lu_0 and $\frac{\partial h}{\partial y}$ (at $0 < t < T$) belong to space B_0 . Suppose there is a constant C such that

$$\left\| \frac{1}{2} u_0 \right\|_p \leq C, \quad \left\| \frac{1}{2} u_{0x} \right\| \leq C. \quad (9)$$

If values $|^n h|$ are continuous with respect to t at $0 < t < T$, operator f is continuous with respect to t . Now we will prove that operator f is quasidifferential, i.e., the following inequality is satisfied at $w_1, w_2 \in B_p$, $p < p_0$,

$$\|f(w_1) - f(w_2)\|_\rho \leq (1 + \frac{\partial}{\partial \rho}) [F(\|w_1\|_\rho + \|w_2\|_\rho) \|w_1 - w_2\|_\rho] \quad (10)$$

with function $F(y)$ of the substantial variable $y \geq 0$ such that $F \geq 0$, $F' \geq 0$, $F'' \geq 0$. We will have

$$\begin{aligned} f(w_1) - f(w_2) = & (-\frac{1}{2} w_1 w_{1n} + \frac{1}{2} w_2 w_{2n}) + (-\frac{1}{2} u_0 w_{1n} + \\ & + \frac{1}{2} u_0 w_{2n}) + (lw_1 - lw_2) + (-\frac{1}{2} u_{0n} w_1 + \frac{1}{2} u_{0n} w_2). \end{aligned}$$

Using (6) and (9), we will estimate the norms of the terms on the right:

$$\| \frac{1}{2} u_{0n} w_2 - \frac{1}{2} u_{0n} w_1 \|_\rho \leq 0 \|w_1 - w_2\|_\rho,$$

$$\| \frac{1}{2} u_0 w_{2n} - \frac{1}{2} u_0 w_{1n} \|_\rho \leq 0 \frac{\partial}{\partial \rho} \|w_1 - w_2\|_\rho,$$

further, we will have

$$\begin{aligned} \|w_2 w_{2n} - w_1 w_{1n}\|_\rho & \leq \|w_2 w_{2n} - w_2 w_{1n}\|_\rho + \|w_2 w_{1n} - w_1 w_{1n}\|_\rho \\ & \leq \|w_2\|_\rho \frac{\partial}{\partial \rho} \|w_1 - w_2\|_\rho + \|w_1 - w_2\|_\rho \frac{\partial}{\partial \rho} \|w_1\|_\rho \end{aligned}$$

and

$$\|w_2 w_{2n} - w_1 w_{1n}\|_\rho \leq \|w_1\|_\rho \frac{\partial}{\partial \rho} \|w_1 - w_2\|_\rho + \|w_1 - w_2\|_\rho \frac{\partial}{\partial \rho} \|w_2\|_\rho$$

whence

$$\|\frac{1}{2} w_2 w_{2n} - \frac{1}{2} w_1 w_{1n}\|_\rho \leq \frac{1}{4} \frac{\partial}{\partial \rho} [(\|w_1\|_\rho + \|w_2\|_\rho) \|w_1 - w_2\|_\rho].$$

We will show that

$$\|lw_1 - lw_2\|_\rho \leq \frac{1}{2} \frac{\partial}{\partial \rho} \|w_1 - w_2\|_\rho.$$

Actually, if

$$w = w_1 - w_2 = \sum_{n=0}^{\infty} \sum_{k=0}^n \frac{1}{k!(n-k)!} \tilde{w}_{k,n-k} x^k y^{n-k},$$

then

$$lw = \frac{1}{2} \int_0^1 \tilde{w}_{yy} dx = \frac{1}{2} \sum_{n=2}^{\infty} \sum_{k=0}^{n-2} \frac{1}{(k+1)!(n-k-2)!} \tilde{w}_{k,n-k} x^{k+1} y^{n-k-2}$$

whence

$$|l w| = \max_{0 \leq k \leq n-2} \frac{1}{2} |\tilde{w}_{k,n-k}| \leq \frac{1}{2} |w|$$

and

$$\begin{aligned} \|\tilde{w}\|_\rho &= \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{n!} |\tilde{w}|_{\rho_n} \\ &\leq \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{n!} |\tilde{w}|_{\rho_n} = \frac{1}{2} \frac{\partial}{\partial \rho} \|\tilde{w}\|_\rho. \end{aligned}$$

The estimates obtained show that in our case, inequality (10) is valid with the function

$$F(y) = c + \frac{1}{2} + \frac{1}{4} y.$$

All the conditions of L. V. Ovsyannikov's theorem are satisfied, and it guarantees that problem (7), (8), and that means also problem (3), (4), have a single solution from space B_ρ for any $\rho < \rho_0$ and sufficiently small $t \geq 0$. More precisely, there exists a $k > 0$, so that the solution belongs to B_ρ for the values of ρ, t from the region

$$\Delta = \{(\rho, t): \rho + kt < \rho_0, t \geq 0, 0 < \rho < \rho_0\}$$

In conclusion, the author is indebted to L. V. Ovsyannikov for his helpful advice.

Bibliography

- [1] C. C. Lin, E. Reissner, H. S. Tsien. J. Math. and Phys., 27, No. 3, 220, 1948.
- [2] O. S. Ryzhov. Study of Transonic Flows in Laval Nozzles. M. VTs AN SSSR, 1965.
- [3] Ye. V. Mamontov, DAN, 185, 3, 538, 1969.
- [4] Ye. V. Mamontov, Coll. "Solid Medium Dynamics," Iss. 1, Novosibirsk, 1969.
- [5] L. V. Ovsyannikov, DAN, 200, 4, 789, 1971.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER FTD-ID(RS)T-1579-77	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ANALYTICAL PERTURBANCES IN AN UNSTEADY TRANSONIC FLOW		5. TYPE OF REPORT & PERIOD COVERED Translation
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ye. V. Mamontov		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Foreign Technology Division Air Force Systems Command U. S. Air Force		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 1972
		13. NUMBER OF PAGES 9
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) 20		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

ORGANIZATION	MICROFICHE	ORGANIZATION	MICROFICHE
A205 DMATC	1	E053 AF/INAKA	1
A210 DMAAC	2	E017 AF/RDXTR-W	1
B344 DIA/RDS-3C	8	E404 AEDC	1
C043 USAMIIA	1	E408 AFWL	1
C509 BALLISTIC RES LABS	1	E410 ADTC	1
C510 AIR MOBILITY R&D	1	E413 ESD	2
LAB/FIO		FTD	
C513 PICATINNY ARSENAL	1	CCN	1
C535 AVIATION SYS COMD	1	ETID	3
C557 USAIIC	1	NIA/PHS	1
C591 FSTC	5	NICD	5
C619 MIA REDSTONE	1		
D008 NISC	1		
H300 USAICE (USAREUR)	1		
P005 ERDA	1		
P055 CIA/CRS/ADD/SD	1		
NAVORDSTA (50L)	1		
NASA/KSI	1		
AFIT/LD	1		